Professor's Page

WHY REASONING?

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Reasoning is one of the proficiency strands of the new Australian Curriculum. Of course, reasoning has always been important in mathematics and its importance has always been recognised in mathematics curricula across Australia. However, the new proficiency strand provides an opportunity for all teachers to reconsider how they teach this essential aspect of mathematics. There are many aspects to reasoning in mathematics, but this focuses on the reasoning that establishes why mathematical results are true.

Mathematics is distinguished amongst the areas of human knowledge by the special way in which claims of what is true are justified. The assumptions (technically called axioms) and definitions are stated, and gradually, piece by piece, all other mathematical knowledge is built up using the rules for logical deduction. It is an enormously complex web, but the consequence is that mathematical results can be definitely proved. This is not true to nearly the same extent for any other subject.

How can this fundamental characteristic of mathematics be conveyed at school?

Consider the observation that an even number plus an even number is odd, that odd plus odd is even and that even plus odd is odd. It is useful to know these even–odd adding rules. After all these years of calculating, I still use them as a quick and virtually automatic check when adding and subtracting: for example if I need to add three numbers and I notice they are all odd, then I know that my answer must be an odd number.

There are several ways in which the even–odd adding rules can be justified in the classroom. One way is to rely on the superior knowledge of the teacher, without other reasons. "My teacher said so, so they are correct." This approach does not help students to develop reasoning or independent mathematical thinking.

Another way is to have children look at many examples: $3 + 5 = 8$, $4 + 6 = 10$, $1 + 7 = 8$ and so on, and then to find the rules for themselves. This has definite advantages—children are actively engaged in making and testing generalisations and can experience the joy of discovering a pattern and sharing it. But in a classroom that values mathematical reasoning, even young children can do more than guess patterns based on evidence from examples. Mathematical truth is not established just by looking at examples.

To uncover the reasons for the even–odd adding rules, we need to understand better what an even number is, not just know that 2, 4, 6, 8 and the rest are called even numbers. In adult language, even numbers are multiples of 2; in the language of small children, even numbers of objects can be put into pairs (groups of two). With odd numbers, the pairing always leaves one left over. The top arrows of Figure 1 illustrate this.

Equipped with this mathematical definition, we can see why the even–odd adding rules must be true. To add an odd and an even number of blocks, we have one set of blocks that consist of pairs, and another that consists of pairs with one left over. When we

add by combining the two sets, we have pairs with one block left over. So even plus odd is odd, as in Figure 1.

To show that an odd number plus an odd number is even involves just one extra step, shown in Figure 2. The two sets of blocks are both made up of pairs with one block left over. After combining the blocks (adding), the two left over blocks can be put together to make another pair. So an odd number plus an odd number is even.

A serious mathematical proof would be written using symbols to stand for generalised numbers rather than illustrated with blocks, but the essential idea of the proof (combining those left over blocks to make another pair) is just the same. Quite young students can see that this idea generalises—that it works for any number of blocks. There is nothing special about the numbers of blocks chosen in the figures.

The example of the even–odd adding rules illustrates several important points:

1. Even very young children can engage in mathematical reasoning, so logical reasoning can and should feature in mathematics classes at all levels.
2. Reasoning should be age appropriate, so in the primary grades it will frequently involve reasoning with models like blocks.
3. As students progress, their reasoning will become more sophisticated across many dimensions. This needs systematic attention throughout the years of school. One dimension for growth is appreciating the nature of the evidence given for why mathematical statements are true. Examples confirm in mathematics, but only a logical argument can provide convincing evidence that a mathematical claim is true for an infinite number of cases.
4. Teachers should not be afraid to explain age-appropriate reasons why mathematical results are true. It is hard to do this well. Students need to be prepared, for example, by gathering evidence (e.g., by measuring) and looking for patterns so that they can guess results and get a sense of discovery.
5. Look at the resources used at your school—the textbooks, worksheets, computer programs or lesson plans. Make sure that there is attention to explaining reasoning.
6. Set tasks that require students to explain their thinking and why their discoveries might be true. Discussion is a good way to clarify reasons before writing them down.
7. Right from the start, students can get to know mathematics as the subject where they do not need to remember rules without reasons. They should not focus on memorising what the teacher says, but know that they can often think things through for themselves. Mathematics makes sense and is a coherent whole.

Wherever possible, give reasons—maybe using a model, as in the case of the even–odd addition rules. Mathematical reasoning at school will not be the same as a mathematician would use—we need explanations appropriate to students' development. However, all mathematics lessons, at whatever level, need to convey the impression that knowing the reasons why mathematical rules are true is important and the key to learning it well.

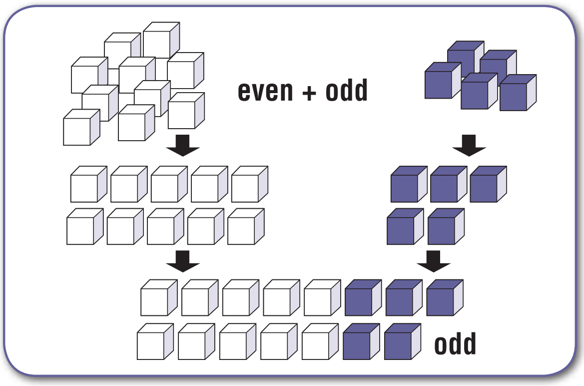

Figure 1. An even number plus an odd number is an odd number.

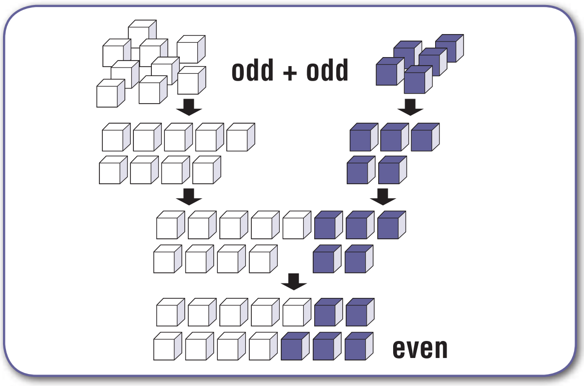

Figure 2. An odd number plus an odd number is an even number.